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Abstract. By resorting to recent results, we show that an isomorphism exist between linguistic features of the Minimalist Program and the quantum field theory formalism of condensed matter physics. Specific linguistic features which admit a representation in terms of the many-body algebraic formalism are the unconstrained nature of recursive Merge, the operation of the Labeling Algorithm, the difference between pronounced and un-pronounced copies of elements in a sentence and the build-up of the Fibonacci sequence in the syntactic derivation of sentence structures. The collective dynamical nature of the formation process of Logical Forms leading to the individuation of the manifold of concepts and the computational self-consistency of languages are also discussed.

1. Introduction

In this paper, by closely following refs. [1], we present some recent results showing that some features of the so-called Minimalist Program [2] (MP) in linguistic studies are quite well suited to a mathematical representation in terms of algebraic methods and tools. The founder and main contributor of the linguistic domain called "Generative Grammar" (GG), has been, and still is, Noam Chomsky (see his contribution to these Proceedings [3]). Details of some specific linguistic features and of the quantum field theory (QFT) mathematical formalism can be found in [1]. For brevity we will omit to discuss the debate concerning the GG approach and its developments. For some basic references see e.g. [4] and [5] where a use-based explanation of language is offered; for counters see [6] and [7, 8]; for a statistical approach to syntax and language learning, see [9, 10, 11], for counters see [12, 13]. See [14, 15] and references therein for a broader approach to language and language evolution.

In our opinion, the results we present are very rewarding since, on the one hand, they allow to recognize the deep dynamical processes underlying the MP linguistic structures; on the other hand, they show the linguistic content of the many-body formalism.

The plan of the paper is the following. The MP and the basic Merge operation are presented in Section 2 (see also the Appendix A). In Sections 3 are discussed the X-bar tree, its self-similarity properties and the breakdown of time-reversal symmetry. The interfaces, the manifold of concepts and the copies of lexical elements, comments on the entropy and the computational self-consistency of language are presented in Section 4. Section 5 is devoted to final remarks. Finally, the definition and properties of the Fibonacci matrix are presented in the Appendix B.

2. The binary Merge and the third factors of language design

In the "The Minimalist Program" an increasing emphasis has been put in recent years on "third factors of language design", namely, principles that are basically minimal, strictly local search, minimal computation, not specific to language, nor specific to biological systems. In other words, the physics and the mathematics of language. The other two factors are genetic predispositions and peculiarities of the local language that the child has to learn [16].

In the early Eighties, the Generative Grammar theory developed in its refined form of the theory of Government and Binding (GB). The core of GB were several syntactic "modules", each taking care of a kind of syntactic operations (Case, Theta Criterion, Binding, Control etc.). See [17] for a basic introduction. It turned out that all Phrases had the recursive structure of a tree: the X-bar tree, or X-bar structure. An element of the structure, a node of the X-bar tree, can contain another X-bar structure, and so on recursively. The symbol X generically denotes all kinds of Phrases. In Minimalism, the picture has been drastically reduced, gaining in simplicity and depth of explanatory power [18].

The basic operation now is binary Merge: combine two elements of the lexicon, α and β, into a binary un-ordered set $\{\alpha, \beta\}$. This is, as a whole, of the same category as one of them (the head). For instance: $\{_\alpha \alpha, \beta\}$. *The smart man* is a man, *man* is the head, and we have a Noun Phrase; *Had been bought* is an event of buying (the verb *buy* in the past tense is the head), we have a Verb Phrase. This set, as a whole, is then Merged with a third element from the lexicon γ getting $\{_\gamma \gamma, \{_\alpha \alpha, \beta\}\}$. The new construct (Syntactic Object, SO) can have the category of γ.

Without going into further details, we observe that this binary Merge of larger and larger syntactic objects is again recursively repeated, in more complex sentences, with subordinates, relatives, embeddings, until the whole sentence is terminated [19]. The syntactic process, called "derivation", is similar to a proof ending when the sentence is terminated.

Merge can create binary sets by pairing different lexical items or whole constituents (e.g. $\{V, NP\}$ a verb and a noun phrase). This is called External Merge (EM). Internal Merge (IM) merges a set X with a term Y of X (where a term is a subset of X or a subset of a term of X), yielding $\{Y, X\}$ with two *copies* of Y, one the copy merged with X and one the term Y of X, which remains, because X is unchanged by the operation. This means that displacement yields copies, providing the basis for semantic interpretation ("reconstruction"). Syntactic movement is now reduced to the operation of IM. Merge leaves the items being merged unaltered (this is called No Tampering Condition; for instance, the sub-units that form a composite word like *undiagonalizable* (*un-, al,- ble*) are never treated separately by the derivation).

Note that there are intermediate cyclic points of derivational (computational) closure, called Phases. We will use upper case P for Phases in syntax and lower case p for phases in physics. Thus, the syntactic derivation (the specific mental computation) stops when a Phase is reached, and then a higher Phase is opened. The process continues inside-out, building higher and higher components in the syntactic hierarchy.

There is also "pair-merge", for asymmetric adjunction, when a sentence has some optional qualifications of the noun phrase (as in *the book of poems [with the glossy cover] [from Blackwell]*, where the expressions in square brackets are adjuncts). This Merge must be asymmetric. We cannot have *the book with the glossy cover from Blackwell of poems*. Finally, we have the Agree operation (*he goes, they go, he has gone, they have gone*) under conditions of minimal, strictly local search. Syntax has no other component than these, it is therefore called Narrow Syntax.

In the previous theory of GB [20, 21], syntax had many operators and modules. In MP, these are subsumed under the basic operation Merge. The head gives the name to the constituent it generates, nouns to Noun Phrases, verbs to Verb Phrases and so on. Generally, we have an $\{H, XP\}$ construction, a Head and a Phrase (the X in XP denoting any phrasal category).

Heads such as nouns, verbs, adjectives, prepositions, have been known in traditional linguistics; other ones, such as complementizer, inflection, tense, negation and more, are of

abstract kind and their identification has been not obvious. What were in GB called "empty categories", because they are not pronounced or written, are now simplified in terms of copies. Copies are elements already present in previous steps of the derivation, for instance items extracted from the lexicon.

We remark that the condition of "strict locality" applies to the structure of the sentence. The number and kind of nodes separating the affected elements in the syntactic tree are important, not the number of words separating the affected elements in the sentence. This is a central property of syntax and is called "structure dependence". It constitutes a sharp departure from anti-generativist approaches to language based on statistics or conventions of use.

3. The X-bar tree, the Fibonacci progression and the breakdown of time-reversal symmetry

In Generative Grammar we have a collection of binary entities. Lexical items are represented by convention as $(+/-)$; for instance Nouns as $(+N, -V)$, Verbs as $(-N, +V)$. This notation can be extended to Phrasal Heads $(+H, -C)$ and Complements $(-H, +C)$. In the syntactic derivation, we have Terminal nodes $(+T)$ and nonterminal nodes $(-T)$. Copies of lexical items, or of larger structures, in a sentence can be pronounced $(+Pr)$ or not-pronounced $(-Pr)$. Recursive applications of Merge may produce a Phase $(+Ph)$ or not $(-Ph)$. The basic syntactic operation, Merge, generates a binary set. Syntactic trees (the X-bar tree) have thus only two branches departing from each node, which is referred to as "binary branching" [22].

We formalize this in terms of vector space of states, the Pauli matrices σ_i, $i = 1, 2, 3$, and the unit matrix I. The vector space on which the matrices operate is built on the basis of the states $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, denoted by $|0\rangle$ and $|1\rangle$, respectively, with scalar product $\langle i|j\rangle = \delta_{ij}$, $i, j = 0, 1$. In full generality, they may represent two lexical elements or two levels of the same lexical element. They are fermion-like eigenstates of σ_3 with eigenvalue $\mp 1/2$, respectively. Thus we consider the $SU(2)$ group and the $su(2)$ algebra [23] with commutation relations which, written in terms of the matrices $\sigma^\pm = \sigma_1 \pm i\,\sigma_2$, are

$$[\sigma_3, \sigma^\pm] = \pm\sigma^\pm, \quad [\sigma^-, \sigma^+] = -2\sigma_3 \,. \tag{1}$$

The Pauli matrices σ_i, $i = 1, 2, 3$, and the unit matrix I form a basis for the $SU(2)$ group, so that any 2×2 matrix can be expressed in terms of superpositions of the σ_i, $i = 1, 2, 3$, and I. For example, the so-called Chomsky matrix C_1 is given by

$$C_1 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} = \sigma_3 + \frac{I}{2} + i\left(\sigma_3 - \frac{I}{2}\right) = \sigma^+\,\sigma^- - i\,\sigma^-\sigma^+ \tag{2}$$

and its hermitian conjugate $C_2 = C_1^\dagger = \sigma^+\,\sigma^- + i\,\sigma^-\sigma^+$.

We will assume to have a collection of N two-level objects ("particles" or "lexical elements"). In a standard fashion they are represented for each i by the "ground states" $|0\rangle_i$ and "excited states" $|1\rangle_i$, $i = 1, 2, 3, ...N$. We also write $\sigma_{3i} = \frac{1}{2}(|1\rangle_{ii}\langle 1| - |0\rangle_{ii}\langle 0|)$, with eigenvalues $\pm\frac{1}{2}$, and $\sigma_i^+ = |1\rangle_{ii}\langle 0|$ and $\sigma_i^- = |0\rangle_{ii}\langle 1|$. In the algebra Eq. (1), we then have $\sigma^\pm = \sum_{i=1}^N \sigma_i^\pm$ and $\sigma_3 = \sum_{i=1}^N \sigma_{3i}$. We recall that the anticommutation relations of the σ's are $\{\sigma_i, \sigma_j\} = (1/2)\delta_{ij}I$, implying the antisymmetry under permutation of the fermion-like states.

In order to see how the "binary Merge" between two states is generated, consider first the two states $|0\rangle$ and $|1\rangle$. In the following we will consider generalization to the collection of N elements and restore the index i, which now for simplicity we omit. Start with $|0\rangle$. Here and in the following we do not consider the (trivial) possibility to remain in the fundamental state $|0\rangle$ (which is dynamically equivalent to "nothing happens"). The physical meaning of this is

that we neglect fluctuations in the ground state, which can be described by $\sigma^- \sigma^+ |0\rangle = 1|0\rangle$, i.e. $|0\rangle \to |0\rangle$. In the quantum formalism, this is achieved by considering the normal ordering or Wick product of the operators. The interesting possibility is the one of the excitation process from $|0\rangle$ to $|1\rangle$. This is obtained by applying σ^+ to $|0\rangle$: $|0\rangle \to \sigma^+|0\rangle = |1\rangle$, which, by associating $0 \leftrightarrow |0\rangle$ and $1 \leftrightarrow |1\rangle$, may be represented as $0 \to 1 \equiv |1\rangle$. As a first single step the state $|1\rangle$ has been singled out. After that we describe the "action" on that state by application of the sigmas.

Consider now that from the anticommutation relations above recalled $\sigma^+ \sigma^+ = 0 = \sigma^- \sigma^-$, which express the Pauli principle, and $\sigma^+ \sigma^- |1\rangle = 1|1\rangle$ and $\sigma^- \sigma^+|0\rangle = 1|0\rangle$. Note that application of $\sigma^+\sigma^-$ is considered to produce a single step since it is equivalent to the application of the unit matrix I to $|1\rangle$. Note also that $|1\rangle \to |0\rangle$ describes the "decay process" of the excited state . $|1\rangle \to |1\rangle$ describes the "persistence" in the excited state, which represents a dynamically non-trivial possibility and thus we have to consider it. We thus may easily obtain the "tree"

$$\nearrow \quad \sigma^- \sigma^+ |0\rangle = |0\rangle \quad \to \quad \sigma^+|0\rangle \quad = \quad |1\rangle \tag{3}$$

$$|0\rangle \quad \to \quad \sigma^+|0\rangle = |1\rangle \quad \to$$

$$\searrow \quad \sigma^+ \sigma^- \sigma^+|0\rangle = |1\rangle \quad \begin{matrix} \nearrow \sigma^- \sigma^+ |0\rangle = |0\rangle \\ \searrow \sigma^+ \sigma^- \sigma^+ |0\rangle = |1\rangle \end{matrix} \tag{4}$$

$$1 \qquad\qquad 1 \qquad\qquad\qquad 2 \qquad\qquad 3 \tag{5}$$

and so on. At each step, new branching points $\Big\langle$ (new nodes of the X-bar tree) are obtained and the X-bar tree is generated by the $SU(2)$ recursive dynamical process.

Note that in general, any power $(\sigma^+\sigma^-)^n \sigma^+|0\rangle = 1 \times \sigma^+|0\rangle$, for any n.

The conclusion at this point is that as a direct consequence of the fermion-like $su(2)$ algebra we have the "number of the states" in these first steps in the sequence: 1 1 2 3, starting with $|0\rangle$, [one state] , then [2 states], and (3) and (4) [3 states], (cf. Eq. (5)).

From here, from the two $|1\rangle$'s we will have in the next step two $|0\rangle$'s and two $|1\rangle$'s, and from the $|0\rangle$ we will get one single $|1\rangle$, in total 5 states: 1 1 2 3 5. We will get thus, in the subsequent steps, other states, and their numbers obtained at each step are in the Fibonacci progression $(\{F_n\}, F_0 \equiv 0)$ with the ones obtained in previous steps. In general, suppose that at the step F_{p+q} one has p states $|0\rangle$ and q states $|1\rangle$; in the next step we will have: $(p+q)|1\rangle$ and $q|0\rangle$, $F_{q+(p+q)}$. In the subsequent step: $(p+2q)|1\rangle$ and $(p+q)|0\rangle$, a total of states $2p+3q = (q+p+q)+(p+q)$, i.e. the sum of the states in the previous two steps, according to the rule of the Fibonacci progression construction.

In conclusion, the X-bar tree (or F tree) is obtained as a result of the $SU(2)$ dynamics, and its recursivity or self-similarity properties turn out to be described by the Fibonacci progression.

It is interesting to remark that $(\sigma^+\sigma^-)^n \sigma^+|0\rangle = 1 \times \sigma^+|0\rangle$ can be thought of as a "fluctuating" process: σ^+ applied to the (ground) state $|0\rangle$ excites it to $|1\rangle$. Then σ^- brings it down to $|0\rangle$, and σ^+ again up to $|1\rangle$: $\sigma^+\sigma^-$ induces fluctuations $|1\rangle \leftrightarrow |0\rangle \leftrightarrow |1\rangle$ (through the "virtual" state $|0\rangle$), this is the meaning of the fact above observed that $\sigma^+\sigma^-$ is equivalent to 1 at any power n when operating on $\sigma^+|0\rangle$. Similar fluctuations can be obtained by considering $(\sigma^-\sigma^+)^n \sigma^-|1\rangle = 1 \times \sigma^-|1\rangle$, i.e. $|0\rangle \leftrightarrow |1\rangle \leftrightarrow |0\rangle$. This "fluctuating activity" corresponds, in the syntactic derivation, to successive applications of Merge. Simplifying a bit, when recursive Merge reaches the topmost node of a Phase, that is, a point of computational closure, everything underneath, in the tree, becomes off limit. The condition called "Phase Impenetrability Condition" [19],[24]-[26] specifies that nothing in a lower Phase is accessible to the syntactic operations that create the immediately higher Phase. The syntactic objects of the lower Phase and the lower Phase itself are dynamically "demoted" to a $|0\rangle$ state. The "fluctuating activity" is also much suggestive when one thinks of the processes (of milliseconds or so) in the selection of lexical items and the recursive Merge of these into syntactic objects.

3.1. Self-similarity, brain functional activity and breakdown of time-reversal symmetry

Note that in the construction of X-bar tree clarifying "how to go" from one step to the next one introduces the "dynamics". We have thus an "operatorial dynamics" by which multiplicities of states are generated. We observe that a Jaynes-Cummings-like model Hamiltonian can be constructed generating the full set of σ^+ and σ^- products of any order, in all possible orderings compatible with the $su(2)$ algebra (the products used above and leading, as we have seen, to the F progression). Thus, the X-bar tree, which plays so a crucial role in the MP, arises as a result of a *dynamical* model in linguistic, its recursive property being related to the self-similarity property of the Fibonacci progression. The paramount importance of the Fibonacci progression in language has been stressed in [27]-[29]. In the Appendix B we introduce the Fibonacci matrix and show its relation with the Chomsky C_1 matrix. We remark that according to known results in QFT [30],[31]-[33], fractal-like self-similar structures, such as the ubiquitous logarithmic spiral, the Fibonacci progression, the golden spiral and other self-similar structures signal the occurrence of underlying coherent dynamical processes, so that these fractal structures appear as the *manifestation* of the coherent states of the system (more specifically of squeezed or q-deformed coherent states). It is also remarkable that in neuroscience, the analysis of the brain functional activity also shows self-similar features [34, 35] again related to the coherent amplitude and phase modulated neuronal oscillatory patterns observed in the laboratory. It appears thus that our discussion of the recursive Merge in MP points to the emergence of a quite general phenomenon which is deeply rooted in the very brain physiological activity. In the following we comment more on this point.

We close this subsection by observing that at any given step of the X-bar tree (the F tree), the simple knowledge of the state $|0\rangle$ or $|1\rangle$ is not sufficient in order to know its parent state in the previous step; we should also know which one is the branch we are on. This in part corresponds to the Phase Impenetrability Condition mentioned above and to one of the major problems in all of contemporary linguistic theory. In speaking and reading we proceed left to right, from the "outside" (the main sentence), to the "inside" (subordinate sentence), but the syntactic derivation proceeds from right to left, from inside out. This creates a conflict, that presumably the construction of Phases, of periodic points of closure, solves [29, 36, 37].

While the tree construction (the "way forward") is fully determined by the σ's operations, the "way backwards" is not uniquely determined solely by the knowledge of the state $|0\rangle$ or $|1\rangle$. On the other hand, suppose one goes backward q steps starting from a given, say, $|1\rangle$ (or $|0\rangle$) . Then returning to such a specific state is no more guaranteed since at each branching point one has to chose which way to go (unless one keeps memory of its previous path, the Ariadne's thread...). In the syntactic derivation, "forward" consists in building further structure from the inside out, from right to left, proceeding upwards in the syntactic tree. The opposite, "backwards", consists in the derivation "looking down" to lower levels. The Phase Impenetrability Condition, as we have just seen, constrains this operation to a strict minimum. Omitting details, only the leftmost (and topmost) "edge" of the lower Phase is (quite briefly) still accessible to the operations building the next higher Phase.

The lesson is that, parameterizing by time the moving over the X-bar tree, time-reversal symmetry is broken. We therefore need to deal with dissipative formalism.

4. Semantics, copies of lexical elements and self-consistency of language

The conceptual interpretive (semantic) system (CI) and the sensory-motor system (articulation, auditory or visual perception) (SM) are assumed to be the two systems with which Narrow Syntax has to interface. Language cannot be simply restricted to "sounds" with meanings, it needs to include gestures as in Sign Language, also touch in the case of deaf-and-blind subjects [38, 39] (see also the analysis of American Sign Language [40] and the Tadoma method for deaf-and-blind subjects [38]).

Although in a highly nonlinear process, the CI "sees" all copies of the lexical elements, and interprets them; however, at the SM interface only one copy is pronounced (externalization) while the other copy (or copies) remains silent (is deleted at SM), as, for instance, in: *Which books did you read [books]?*. In most languages only the higher copy is pronounced, but there are languages in which the lower copy is pronounced and also languages in which all copies are pronounced. In the latter case, this applies to "short" elements (equivalent to the English "who", "which" and similar), never to whole Noun Phrases. We thus realize that in linguistics "copies" are important objects. We will show below how our modeling may account for this.

The present view is that Merge works without categorial constraints (a bit like Feynman's sum of all histories, before amplitudes give the wave function). With some simplifications, we can say that categories are needed only at the interface with CI for interpretation: which one is a verb, which one a noun, an adjective etc. (labeled heads). Thus, a minimal search process, or labeling process, *The Labeling Algorithm* [41]-[43], has to produce such a categorization at the CI interface. Ordering, implying non-commutativity, is, however, also important. There are two notions of order to be taken into account: ordering of the syntactic operations, which will be considered in the following, and ordering of the items in the externalized linguistic expression at the Sensory-Motor interface, namely, what to pronounce first, second etc. and what not to pronounce at all (deleted copies). Such an ordering, which needs to preserve anyway the interpretative categorization, is probably a reflex of the Sensory Motor system, not feeding narrow syntax or CI. Also in relation to previous views and analyses, see [44]-[48].

In summary, Third Factors (physics), underlying unconstrained Merge, the Labeling Algorithm and general principles of Minimal Computation (MC), are thought to account for all syntax properties. We show that this finds an explicit formalization in terms of QFT, with the absolutely not trivial result that linguistic features are actually manifestations of underlying dynamical processes.

4.1. From syntax to semantics. The manifold of concepts

Let us resume the discussion of the algebraic formalism. We restore the subscript i labeling the N elements introduced in Section 3. The associated states can be regarded as the ones at a given step, of high multiplicity, in the Fibonacci tree. Since N can be as large as one wants, we may always have a direct product of a large number (in principle an infinite number, hence one needs field theories) of factor states, $\Pi_{i=1,N}|s_i\rangle \equiv |s_1\rangle \otimes |s_2\rangle \otimes ...|s_i\rangle \otimes ... \equiv |s_1, s_2, ...s_i...\rangle$, with $s_i = 0$ or 1 for each $i = 1, 2, .., N$. The most general state, denote it by $|l\rangle$, is then a superposition of all states with l elements in $|1\rangle$ and $N - l$ elements in $|0\rangle$. The difference between the number of elements in $|1\rangle$ and the one of the elements in $|0\rangle$ is measured by σ_3 and is given by $\langle l|\sigma_3|l\rangle = l - \frac{1}{2}N$. This quantity is called the order parameter. Its being non-zero signals that the $SU(2)$ symmetry is broken (like, for example, in the case of the magnetic dipoles, where the order parameter provides the measure of the magnetization).

One can show that in the large N limit the $su(2)$ algebra of the σ matrices, represented in the space of the $|l\rangle$ states, for any l, and written in terms of $S^\pm$ and $S_3 \equiv \sigma_3$, where $S^\pm = \sigma^\pm/\sqrt{N}$, "contracts" ("rearranges") into the Weyl-Heisenberg algebra

$$[S_3, S^\pm] = \pm S^\pm, \quad [S^-, S^+] = 1. \tag{6}$$

The result Eq. (6) is a central result. It expresses the "rearrangement" of the $su(2)$ algebra [Eq. (1)] in the $e(2)$ algebra [Eq. (6)], which is isomorph to the Weyl-Heisenberg algebra, with S_3 playing the role of the number operator and $S^\pm$ the role of *boson* ladder operators. Notice the dynamical transition from fermion-like degrees of freedom in the $su(2)$ algebraic frame above discussed to the Weyl-Heisenberg ($e(2)$) boson algebra. Such a *dynamical rearrangement of symmetry* and the contraction of the algebra for large N are well known

dynamical processes in QFT [33],[49]-[52]. They occur when there is spontaneous breakdown of symmetry characterized by the order parameter, which behaves as a non-vanishing classical field. One can show [33, 49, 52, 53] that then the space of the states of the system splits into (infinitely many) unitarily inequivalent representations of the algebra Eq. (6), that is it undergoes a process of *foliation*, splitting in many physically inequivalent subspaces, each one labeled by a specific value assumed by the order parameter. Each of these subspaces is a well defined vector space and represents a possible *phase* in which our system can live. Moreover, each of these *phases* is characterized by collective, coherent waves, represented by the ladder operators $S^{\pm}$. More specifically, the theorem can be proven [54], which predicts the dynamical formation of long range correlation modes (the Nambu-Goldstone (NG) modes) when the symmetry gets spontaneously broken. In Generative Grammar the symmetry breaking phenomenon (the anti-symmetry of syntax, and the dynamic anti-symmetry of syntax) have been cogently argued for by Richard Kayne [55] and Andrea Moro [56]. This is in part why issues about the status of X-bar (as part of Narrow Syntax or as an emergent configuration of recursive binary Merge) have been recently debated [41, 42] (see also [27]). These NG collective modes $S^{\pm}$ are the carrier of the *ordering* information through the system volume [33, 49, 52, 53]. *Order* thus appears as a collective dynamical property of the system which manifests in the limit $N \gg l$. The order parameter provides indeed a measure of the system ordering. Different degrees of ordering corresponding to different values of the order parameter in a convenient range of variability self-consistently defined by the dynamics (different densities of the NG modes). Different values of the order parameter thus denote different, i.e. inequivalent, phases of the system [33, 49, 52, 53]. The dynamics of the system undergoing *phase transitions* is thus characterized by *criticality*, in much a similar way as the brain functional activity is characterized by critical transition through many regimes in a far from the equilibrium dynamics [34, 35, 57, 58].

We thus realize that our system has undergone a formidable dynamical transition, moving from the regime of being a collection of (fermion-like) *atomistic* components (lexical elements) to the regime of (boson-like) collective, coherent $S^{\pm}$ fields. Our main assumption at this point is to identify a specific *conceptual, meaningful* linguistic content (a Logical Form, LF [59]) with the collective coherent phase associated to a specific value of the order parameter. The semantic level (the "manifold of concepts") thus emerges as a dynamical process out of the syntactic background of lexical elements, in a way much similar (mathematically isomorph) to the one by which macroscopic system properties emerge as a coherent physical phase out of a collection of elementary components at a microscopic (atomistic) level in many-body physics [33, 52].

In conclusion, we can now give a quantitative characterization of the "interfaces" where the Narrow Syntax has to make contact with the Conceptual Interpretative (CI) system: interfaces are met when the spontaneous breakdown of symmetry is met, i.e. as the limit $N \gg l$ is approached. It is there that a specific meaning or "concept" arises by selecting out one representation of the algebra from many of them "unitarily inequivalent" among themselves (each corresponding to a different concept) [57]. The concept appears at that point as a collective mode, not a result of associative process pulling together bits and little lexical pieces, words etc.. Rather it binds all of them together in the specific explicit entanglement of which it is expression, as we will exhibit below. The collectiveness comes from the "phase coherence", whose carriers are the collective NG $S^{\pm}$ fields. It can be shown that coherence guaranties also the stability of the semantic content, its robustness against "defects" at the atomistic level. For instance, missing or partially corrupted, or deformed words, a non perfect grammatical construction, etc. barely affect the "meaning", provided however they occur far from a "criticality threshold". In the opposite case, even a slight change at the elementary level may trigger a phase transition process leading to a "dramatic" (i.e. unitarily inequivalent, in a formal sense) change in the semantic landscape. A measure of the departure from the "semantic stability" is provided by the consistency of the violation of the minimization of the free energy [1]. We also understand

why "only at the interfaces the issues of ordering become relevant". Order indeed is lack of symmetry and it can only appear when this is spontaneously broken.

Categorization and non-commutativity (and order) are only necessary at the CI interface. Indeed, only at the large N limit CI needs labeled heads: which one is a verb, which one a noun, an adjective etc.. We have seen that the formal construction of the binary Merge does not require labeled structures (Noun, Verb, Adjective, Preposition etc.). The necessity of labeling (*The Labeling Algorithm*) only arises at the interface with meaning. Interpreting a Determiner Phrase (*the dog, many boys, most books*) or a Verb Phrase (*was going to Rome, had bought the book*) or a Complementizer Phrase (*that I know him, whether it's wise, who did that*) is a necessity for the Conceptual Intentional system, with the formal label of a syntactic object triggering different Intentional landscapes. Once the Narrow Syntax has made contact with CI system, through the action-perception cycle [57] of the cortex dynamics, the sensory-motor system (SM) gets also involved and therefore the linguistic structures can be externalized, allowing to communicate to other speakers all the required subtleties of meaning.

The formalism here presented thus endorses Chomsky's thesis that Merge is unconstrained, and that issues of labeling (headedness, categorization of lexical items) and ordering only arise at the interfaces of Narrow Syntax with the Conceptual-Intentional (CI) system and the Sensory-Motor (SM) system.

4.2. Copies of lexical elements

We now consider the feature of the copies of lexical elements in the Minimalist Program (MP).

At the end of Section 3.1, we have observed that time-reversal symmetry is broken moving along the X-bar tree. Time-reversal symmetry breakdown characterizes the dynamics of dissipative systems, which are systems open to the environment in which they are embedded. We extend therefore to the X-bar structures the formalism which is used for the study of dissipative systems. In such a mathematical formalism a characteristic feature is the doubling of the system degrees of freedom $A \to \{A, \tilde{A}\}$ [60], where A generically denotes the degree of freedom of the system and $\tilde{A}$ its "copy" or the doubled degree of freedom. They belong to two copies of the algebra $\mathcal{A}$. Thus the doubling is obtained by introducing the mapping $\mathcal{A} \to \mathcal{A} \times \mathcal{A}$ [61]. Typically A and $\tilde{A}$ are operators represented by some matrices, with some properties specified by use of a subscript $\mathbf{k}$, e.g. $A_{\mathbf{k}}$, which, however, we will omit for notational simplicity, as far as no misunderstanding occurs.

We also have the doubling of the state space $\mathcal{F} \to \mathcal{F} \times \tilde{\mathcal{F}}$. The operators A and $\tilde{A}$ act on $\mathcal{F}$ and $\tilde{\mathcal{F}}$, respectively. In the case of dissipative systems, $\tilde{A}$ represent the environment degrees of freedom and the system elements cannot be exchanged with the elements of the environment. In the present case of linguistics, at the syntactic and semantic levels, lexical elements and conceptual contents cannot be simply interchanged. This is described by the algebraic structure called the non-commutative Hopf algebra or q-deformed Hopf algebra, with q called the deformation parameter.

For notational simplicity from now on we will denote by A and $A^{\dagger}$ the operators S^- and S^+ in Eq. (6), respectively. Thus, the doubling process implies that correspondingly we also have $\tilde{S}^-$ and $\tilde{S}^+$, which will be denoted as $\tilde{A}$ and $\tilde{A}^{\dagger}$, respectively.

By adopting $q(\theta) = e^{\pm\theta}$, one can show that the system ground state is [33, 52, 60]

$$|0(\theta)\rangle_{\mathcal{N}} = e^{i\sum_{\mathbf{k}} \theta_{\kappa} G_{\mathbf{k}}}|0\rangle = \prod_{\mathbf{k}} \frac{1}{\cosh \theta_{\kappa}} \exp(\tanh \theta_{\kappa} A_{\kappa}^{\dagger} \tilde{A}_{\kappa}^{\dagger})|0\rangle, \qquad (7)$$

where the subscript $\mathbf{k}$ has been restored, and $|0\rangle \equiv |0\rangle \times |0\rangle$ is the state annihilated by A and $\tilde{A}$: $A|0\rangle = 0 = \tilde{A}|0\rangle$ (the vacuum state). However, A and $\tilde{A}$ do not annihilate $|0(\theta)\rangle_{\mathcal{N}}$. It is annihilated instead by the operators $A(\theta)$ and $\tilde{A}(\theta)$, obtained as

$A(\theta) = \exp(i \sum_{\mathbf{k}} \theta_\kappa G_{\mathbf{k}}) A \exp(-i \sum_{\mathbf{k}} \theta_\kappa G_{\mathbf{k}})$ and $\tilde{A}(\theta) = \exp(i \sum_{\mathbf{k}} \theta_\kappa G_{\mathbf{k}}) A \exp(-i \sum_{\mathbf{k}} \theta_\kappa G_{\mathbf{k}})$. The meaning of the subscript $\mathcal{N}$ is clarified below, θ denotes the set $\{\theta_\kappa, \forall \mathbf{k}\}$ and $|0(\theta)\rangle_{\mathcal{N}}$ is a well normalized state: $_{\mathcal{N}}\langle 0(\theta)|0(\theta)\rangle_{\mathcal{N}} = 1$. The vacuum $|0(\theta)\rangle_{\mathcal{N}}$ turns out to be a generalized $SU(1,1)$ coherent state of condensed couples of A and $\tilde{A}$ modes [23, 60], which are entangled modes in the infinite volume limit. The operator $\exp(i \sum_{\mathbf{k}} \theta_\kappa G_{\mathbf{k}})$, with $G_{\mathbf{k}} \equiv -i(A_{\mathbf{k}}^\dagger \tilde{A}_{\mathbf{k}}^\dagger - A_{\mathbf{k}} \tilde{A}_{\mathbf{k}})$, is the generator of the state $|0(\theta)\rangle_{\mathcal{N}}$ as shown in Eq. (7).

Incidentally, we observe that in language, in first approximation, the vacuum state is silence. Just like in the present algebraic formalism, there are many kinds of silence. Not only how a silence gap is interpreted in the unfolding of a conversation, but in a more specific and more technical sense. There is, literally, a syntax of silence [62] in linguistic constructions called ellipsis (*Mary bough a book and Bill did ____ too*) and sluicing (*Ann danced with someone but I do not know who ____*). Jason Merchant and other syntacticians and semanticists have shown that what can be omitted is never just an arbitrary "bunch of words", but an entire syntactic constituent (an entire Verb Phrase, most frequently). The syntax and semantics of several well defined but unpronounced elements has been part of the theory since the beginning of Generative Grammar [63].

One can show [60, 61, 64] that $_{\mathcal{N}}\langle 0|0(\theta)\rangle_{\mathcal{N}} \to 0$ and $_{\mathcal{N}}\langle 0(\theta')|0(\theta)\rangle_{\mathcal{N}} \to 0$, $\forall \theta \neq \theta'$, in the infinite volume limit $V \to \infty$. Thus we conclude that the state space splits in infinitely many physically inequivalent representations in such a limit, each representation labeled by a θ-set $\{\theta_{\mathbf{k}} = \ln q_{\mathbf{k}}, \forall \mathbf{k}\}$. This is the $q(\theta)$-*foliation* process of the state space. In the present case of linguistics this represents the process of generation of the manifold of concepts. It is a dynamical process since the generator $G_{\mathbf{k}}$ is essential part of the system Hamiltonian [60]. In our present case, "physically inequivalent" representations means that the "manifold of concepts" is made of "distinct", different spaces, each one representing a different "concept" (in language we have the Logical Forms (LFs) composing the global Logical Form of the entire sentence), here described as the coherent collective mode generated through the X-bar tree. These spaces (concepts) are protected against reciprocal interferences since the spaces are "unitarily inequivalent", i.e. there is no unitary operator (here is rooted the criticality of the dynamics) able to transform one space in another space [57, 58], which corresponds to the fact that syntactic Phases cannot be commingled, nor "reduced" one into the other. Phases are mutually impenetrable.

In practice, however, the unitary inequivalence is smooted out by realistic limitations, such as, for example, the impossibility to reach in a strict mathematical sense the $V \to \infty$ limit (i.e. the "infinite number" of lexical elements or the theoretically infinite number of choices for the co-referentiality indices in the logical form of even the simplest sentences). Thus, realistically, we may also move from concept to concept in a chain or trajectory going through the manifold of concepts [34, 35, 57],[65]-[67]. This corresponds to the compositionality of meanings, when the syntactic derivation proceeds "upwards" (that is: forward) from the lower Phases to the higher Phases, from local Logical Forms to the composition of more inclusive Logical Forms. Remarkably, these trajectories in the unitarily inequivalent spaces (the "manifold of concepts") have been shown to be classical chaotic trajectories [65], which again links these processes in language to the chaotic features of brain functional dynamics observed in neuroscience labs and discussed since long in the literature [68].

4.3. Self-consistency of language and its built-in self-reference

In order to better understand the role played by the "tilde copies", $\tilde{A}$, let's compute $N_{A_{\mathbf{k}}} = A_{\mathbf{k}}^\dagger A_{\mathbf{k}}$ in the state $|0(\theta)\rangle_{\mathcal{N}}$

$$\mathcal{N}_{A_{\mathbf{k}}}(\theta) \equiv {}_{\mathcal{N}}\langle 0(\theta)|A_{\mathbf{k}}^\dagger A_{\mathbf{k}}|0(\theta)\rangle_{\mathcal{N}} = {}_{\mathcal{N}}\langle 0(\theta)|\tilde{A}_{\mathbf{k}}(\theta)\tilde{A}_{\mathbf{k}}^\dagger(\theta)|0(\theta)\rangle_{\mathcal{N}} = \sinh^2 \theta_{\mathbf{k}}. \qquad (8)$$

From this we see that for any $\mathbf{k}$ the only non-vanishing contribution to the number of non-tilde modes $\mathcal{N}_{A_{\mathbf{k}}}(\theta)$ comes from the tilde operators, which can be expressed by saying that these last

ones constitute the dynamic *address* for the non-tilde modes (the reverse is also true, the only non-zero contribution to $\mathcal{N}_{\tilde{A}_{\mathbf{k}}}(\theta)$ comes from the non-tilde operators).

This also shows that the physical content of $|0(\theta)\rangle_{\mathcal{N}}$ is specified by the $\mathcal{N}$-set $\equiv \{\mathcal{N}_{A_{\mathbf{k}}}(\theta), \mathcal{N}_{A_{\mathbf{k}}}(\theta) = \mathcal{N}_{\tilde{A}_{\mathbf{k}}}(\theta), \forall \mathbf{k}\}$, called the *order parameter*. The $\mathcal{N}$-set characterizes the vacuum; this also explains the meaning of the $\mathcal{N}$ subscript in $|0(\theta)\rangle_{\mathcal{N}}$.

All of this sheds some light on the relevance of "copies" in the MP. In some sense they are crucial in determining (indeed providing the address of) the whole conceptual content of the considered linguistic structure. They provide the *dynamic reference* for the non-tilde modes. Unpronounced copies, being silent, do not reach the Sensory Motor system, but they are crucially interpreted by the Conceptual Intentional system. They are necessary to the understanding of the meaning of what is actually pronounced. Remarkably, they are "built in" in the scheme here proposed; they are not imposed by hand by use of some constraint "external" to the linguistic system. It is in this specific sense that we speak of "self-consistency": our formal scheme is computationally (logically) self-contained [69]. Perhaps the real power of the linguistic tool available to humans consists in such a specific feature [1].

The entanglement between the modes A and $\tilde{A}$ is seen by expanding $|0(\theta)\rangle_{\mathcal{N}}$ in Eq. (7) as

$$|0(\theta)\rangle_{\mathcal{N}} \;=\; \left(\prod_{\mathbf{k}} \frac{1}{\cosh \theta_k}\right) \left[|0\rangle \otimes |0\rangle + \sum_{\mathbf{k}} \tanh \theta_k |A_{\mathbf{k}}\rangle \otimes |\tilde{A}_{\mathbf{k}}\rangle + \dots\right] \tag{9}$$

and we see that it cannot be factorized into the product of two single-mode states $|A_{\mathbf{k}}\rangle$ and $|\tilde{A}_{\mathbf{k}}\rangle$. A and $\tilde{A}$, for any $\mathbf{k}$, are thus entangled modes. $|0(\theta)\rangle_{\mathcal{N}}$ can be also written as [52, 60]:

$$|0(\theta)\rangle_{\mathcal{N}} = \textstyle\sum_{n=0}^{+\infty} \sqrt{W_n}\,(|n\rangle \otimes |n\rangle),$$

with $W_n = \prod_{\mathbf{k}} \sinh^{2n_{\mathbf{k}}} \theta_k / \cosh^{2(n_{\mathbf{k}}+1)} \theta_k$ and n denoting the set $\{n_{\mathbf{k}}\}$, $0 < W_n < 1$, $\sum_{n=0}^{+\infty} W_n = 1$. The probability of having the component state $|n\rangle \otimes |n\rangle$ in the state $|0(\theta)\rangle_{\mathcal{N}}$ is thus W_n, which would be suppressed for large n in the case of a finite number of terms in the summation, since it is a decreasing monotonic function of n. However, this is not the case if the summation is over infinite terms. In such a limit there is no unitary generator able to disentangle the $A - \tilde{A}$ modes, $|0(\theta)\rangle_{\mathcal{N}}$ and $|0\rangle$ are unitarily inequivalent states. The entropy S is given by [33, 52, 60]

$$_{\mathcal{N}}\langle 0(\theta)|S|0(\theta)\rangle_{\mathcal{N}} = \sum_{n=0}^{+\infty} W_n \, \log W_n \; . \tag{10}$$

Consistently with the breakdown of time-reversal symmetry (the *arrow of time*), one finds that variations of the entropy control the time evolution (the trajectories) in the manifold of concepts (the space of the *infinitely many* LF). In this formalism entropy is thus related with the semantic level of the LF, *meanings*, dynamically arising as collective modes out of the syntactic (atomistic) level of lexical elements. One can also exhibit the free energy functional and show that its minimization is related to the stability of the system dynamic regime (phase) [1].

5. Concluding remarks
In this paper we have reported some recent results showing that the many-body formalism may describe central features of the Minimalist Program in linguistics, with particular reference to the generation of the X-bar tree structure, its self-similarity properties and relation with the Fibonacci progression and the breakdown of time-reversal symmetry. We have discussed the role of the copies in the conceptual interpretative system CI and shown that they can be accounted by the Hopf algebra structure of the doubled tilde operators in the quantum field theory formalism. What emerges from our physical modeling of the Minimalist Program is that a truly dynamical

structure underlies linguistic features. Among others, two specific results may be worth to be stressed. One is the dynamical emergence of Logical Forms, namely the dynamical transition from a numeration of lexical items to syntax and from syntax to the logical form (LF) of the sentence and to meaning. This has brought us to the identification of the "manifold of concepts", formally represented by the set of unitarily inequivalent state spaces in the foliation process of QFT. The formation of concepts appears then as the dynamical generation of collective coherent modes spanning as waves the lexical items entering the sentence or collection of sentences. These collective modes are associated to the ordering of the lexical items, but not strictly dependent on each of them. This aspect is expression of a stability principle, namely the robustness of meaning against corruption, alteration or even deletion of one or more, however below some critical threshold, of the lexical items. On the other hand, different ordering (different meanings), i.e. different values of the order parameter, denote inequivalent phases of the system [33, 49, 52, 53]. The undergoing dynamics of *phase transitions* is characterized by *criticality*: in a similar way as in critical dynamics of the brain functional activity [34, 35, 57, 58], we may have trajectories going through the manifold of concepts [34, 35, 57],[65]-[67] (move from concept to concept). These trajectories have been shown to be classical chaotic trajectories [65]. This again links these processes in language to the observed chaotic features of brain functional dynamics [68].

The other result is the linguistic computational self-consistency. This arises from the nonlinear interaction between the doubled entities (the non-tilde and the tilde modes representing the lexical items and their copies) entering the specific Hopf algebraic structure of the QFT formalism. The copies or tilde modes have been recognized to provide the *dynamical reference* (the "address") of the non-tilde modes. The result is the logical self-consistency, by inclusion of the *reference* terms, of languages. The resulting state is an entangled state of the non-tilde/tilde modes. Within such a scheme we can consistently define thermodynamic operators such as the entropy and the free energy. Languages appear thus to posses in their own structure, a "built in" "truth (or falsehood) evaluation function", a feature where perhaps resides, as we have observed, the real power of the linguistic tool available to humans.

Summing up, we have uncovered the isomorphism between the linguistic aspects of the Minimalist Program and the physics of many-body systems. As observed in [1], although the algebraic properties of the QFT formalism have been exploited by us, in our scheme the linguistic structures are "classical" ones. This should not be surprising, since [33, 52] QFT describes not only the quantum world of elementary particles and condensed matter physics, but also macroscopically behaving systems characterized by ordered patterns [33, 52].

Our analysis also suggests that the dynamics underlying the biochemical phenomenology of the brain behavior [34, 35, 57, 58, 67] may also provide the basic mechanisms of linguistics. As observed, it is remarkable that the brain functional activity also shows self-similar features [34, 35] related to coherent amplitude and phase modulated neuronal oscillatory patterns. This suggests that the recursive Merge in linguistic MP might be the manifestation of a quite general phenomenon, deeply rooted in the very same brain physiological activity.

Appendix A. A little bit of history
(1) *Phrase Structure Grammars*
   Until the mid-Fifties, a "grammar" was the result of applying some procedures (typically,

segmentation and categorization) to a corpus of data[1]. Chomsky's first attempt at a new perspective was his master thesis [71, 72], but did not reach any audience until *Logical Structure of Linguistic Theory* (1955), his "Three models" paper in 1956 [73] and *Syntactic Structures* in 1957 [74, 75]. Before the advent of his "transformational grammar", the dominant models were Phrase Structure Grammars (PSG) a usually long list of general rules specifying how each kind of Phrase (each phrasal constituent) was to be expanded, all the way to the manifest expression of a complete sentence. The arrows indicate "rewriting rules", that is: what appears on the left has to be rewritten as what appears on the right. For instance (a very simple one) for a whole sentence (S), we typically have as constituents a Noun Phrase (NP), a Verb Phrase (VP) and (optionally) a Prepositional Phrase (PP). More concretely

   NP1 → the cat
   AUX → is
   VP → (is) chasing
   NP2 → a mouse
   PP → in the garden
   (S) = The cat is chasing a mouse in the garden.

(2) *Enter Transformational Grammar*

   Chomsky did persuasively show that such grammars were radically insufficient to account for real languages. That there was a *kernel* sentence that can be *transformed* into variants that are intimately related, by permuting the order of the words in the phrases (obtaining passives, interrogatives, negations etc.). In his work *The Logical Structure of Linguistic Theory* LSLT [63], which really was also the first complete phrase structure grammar, kernel sentences were derived by obligatory transformations from a more abstract phrase structure. E.g., a transformation was required to convert [[be-ing] chase] to [be chasing] (and another similar one to yield is), what Haj Ross later called "Affix-hopping". Suitable mathematical analyses, in terms of the Theory of Automata and the theory of recursive functions, did show that Transformational Grammars (TG) constitute a more powerful kind of automata than Phrase Structure Grammars [76]. The only empirically significant concept in Generative Grammar is strong generative capacity. Canonical examples of transformations are like the following:

   The cat is not chasing a mouse in the garden. NEGATION
   A mouse is being chased by the cat in the garden. PASSIVE
   What/who is the cat chasing in the garden? ACTIVE INTERROGATIVE
   What/who is being chased by the cat in the garden? PASSIVE INTERROGATIVE
   What/who is not being chased by the cat in the garden? NEGATIVE INTERROGATIVE

   Instead of having many more ad hoc phrase structure rules, transformations took care of such cases[2]. The kernel, then was generalized into a deep structure distinct from the surface structure. The link with meaning was supposed to be the deep structure, not the surface structure. All speakers understand that the basic syntactic and semantic relations are preserved in sentences such as

   He met his friend.
   He succeeded in meeting his friend.
   He failed to meet his friend.
   He avoided meeting his friend.

   Separate ad hoc phrase structure rules would have to be introduced, while basic transformational rules cover all such construction and explain why the basic meaning is invariant

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[1] The most sophisticated development procedures were those of Zellig S. Harris (Chomsky's teacher, at the time) [70].

[2] Omitting inessential details and stressing that negation has a representation in formal logic, all these sentences have the same Logical Form (provided we insert or do not insert the logical symbol for negation, as required).

at the level of the deep structure[3]. Also, there are physically absent, but mentally present, syntactic elements that PSG[4] rules could not possibly deal with. Between square parentheses what is not pronounced (or written)

He saw the man standing at the bar.
He saw the man [the man was] standing at the bar.
They saw the prisoner escape from the prison.
They saw the prisoner [when they saw the prisoner he was] escap(ing) from the prison.

Importantly, there are verbs that block such constructions[5]. Ill-formed sentences are, by a long tradition, preceded by an asterisk.

They suspected that the prisoner would try to escape.
*They suspected the prisoner trying to escape.

Contrast this with the perfectly OK sentence: *They saw the prisoner trying to escape.*

So, silent elements were introduced for the first time in linguistics. They are silent ("empty" in the technical terminology) but mentally present and very important in the syntactic structure and in the interpretation.

(3) *The Theory of Government and Binding*

In subsequent years, syntactic movement became a key notion, more general than transformations. Elements in a sentence can be moved to a different position, leaving a trace (an "empty category"). The interpretive system understands the meaning of the sentence by (mentally, silently) paying attention to the position of the trace and to what "governs" the trace: *Which books did you read?*

Books are the object of read and are so interpreted, as being to the right of read, where the trace is. Making the trace explicit by the letter t, we have: *Which books did you read t?*

Making syntactic movement even more explicit, we can write:

$$Which \quad books\ did\ you\ read\ t?$$
$$\uparrow\!\leftarrow--\leftarrow--\leftarrow--\leftarrow--\downarrow$$

Several kinds of un-pronounced (and un-written) syntactic elements were introduced, called "empty categories" (empty because un-pronounced). But they had to be governed in a very similar way to the explicit elements of the sentence (this was called ECP, the Empty Category Principle).

Government was exactly defined, but we will not go into it here, because this notion has been subsumed by a deeper, more abstract notion in the current Minimalist version of the theory.

It's worth noticing that there are "barriers" to syntactic movement, that is: some instances of syntactic movement are blocked. There are "islands" to extraction and movement.

That the police would arrest several rioters was a certainty.
*Who was that the police would arrest a certainty?

In the Theory of Government and Binding, these barriers for movement and the nature of "islands" were explained in terms of strict locality. Summarizing and simplifying drastically a number of acute analysis and several refinements, we can indicate here the so-called Minimal Link Condition [2]. Derived from general principles of derivational economy, it stated that movement can only occur to the closest potential landing site.

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[3] We will not detail here how modifier verbs like "succeed" and "fail" are easily expressed in Logical Form.

[4] Phrase Structure Grammars

[5] Lexical semantics explains why these differences exist. Simplifying drastically, verbs like "suspect" (called Psych Verbs) cannot take a gerundive form as their direct object, while verbs of perception like "see" can.

A similar situation occurs, for instance, with agreement. In Italian there are masculine and feminine features for inanimates: *la bella sedia*, [FEM] *il grande coltello* [MASC], also plural/singular for determiners and adjectives (*le belle sedie, i grandi coltelli*). Once the noun gives the interpretable information singular (only one) or several (plural), these agreement features on the determiners and the adjectives are redundant (in fact English does not have them). For reasons of economy, Narrow Syntax must delete all un-interpretable features *before* the sentence reaches the Conceptual-Intentional interface. Basically, the process was the following: uninterpretable features must be *checked* (sic) to be deletable. Syntactic Objects with such features must "check" them with Syntactic Objects that have interpretable features. The search is *strictly local*, as local as possible. Best: inside the same constituent, at worst: inside the same Phase. The process is called probe-goal. When the probing Syntactic Object encounters the goal, the uninterpretable features are deleted and the result is sent to the Conceptual-Intentional interface. If the local search fails, we have an ungrammatical sentence (the derivation is said to "crash"). Uninterpretable features that have been successfully checked are deleted at the Conceptual-Intentional interface, but not necessarily (in fact usually not) also at the Sensory Motor interface. In fact we do see and hear them at the level of pronunciation and writing. In *Le molte belle figurine* (the many beautiful little cartoons) feminine and plural features are visible and audible in the determiner, the quantifier and the noun. This is called Agreement, the object of many, many linguistics studies [77, 78]. Agree is what makes determiners in languages such as Italian (*il, lo , la, molto, molti, poco, pochi* etc.) and adjectives (*bello, bella, belli*) agree with the noun. Also what makes auxiliaries (*have, be*) and past participles agree with the tense of the verb (*era stato visto, furono visti, saranno stati comprati* etc.). Agree must be local, very local, in a structural sense, not necessarily "local" in terms of proximity of words in the surface structure. *I libri di cui mi parlasti erano stati tutti comprati in libreria* (the books you had told me about have all been bought in a bookshop). *Libri* and *comprati* are not "near" on the surface, but they are near structurally, because the Prepositional Phrase *di cui mi parlasti*, which contains a Verb Phrase, veers off (so to speak) into a sideway structure (it's an adjunct). A similar constraint applies to the assignment of Case (nominative, accusative, genitive etc.) [77]. Case assigners (verbs and prepositions) must be structurally local to the nouns, adjectives etc. to which they assign case. Again, locality is structural, not necessarily on the surface. (Let's think of Latin, Russian etc. where Case is overtly expressed).

Appendix B. On the Fibonacci Matrix
We define the Fibonacci matrix [1] to be given by

$$F \equiv \frac{1}{2} I + \sigma_3 + 2\,\sigma_1 = \left(\begin{array}{cc} 1 & 1 \\ 1 & 0 \end{array} \right)$$

For the n-powers F^n of the F matrix, with $n \neq 0$, we have

$$F^n \;=\; \left(\begin{array}{cc} F_{n+1} & F_n \\ F_n & F_{n-1} \end{array} \right) = F_{n-1}\,I + F_n\,F, \qquad \text{n} \neq 0$$

where the matrix elements F_{n+1}, F_n, F_n, F_{n-1}, with $F_0 \equiv 0$, for any $n \neq 0$, are the numbers in the Fibonacci progression $F_0 = 0, F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 5, F_6 = 8, F_7 = 13, \ldots$ Moreover, also the coefficients of the matrices I and F in the last member on the r.h.s. of the above relation are the Fibonacci numbers F_{n-1} and F_n. We can indeed verify that

$$F^1 \;=\; \left(\begin{array}{cc} 1 & 1 \\ 1 & 0 \end{array} \right) = F, \qquad\qquad F^2 = \left(\begin{array}{cc} 2 & 1 \\ 1 & 1 \end{array} \right) \;= I + F,$$

$$F^3 \;=\; \left(\begin{array}{cc} 3 & 2 \\ 2 & 1 \end{array} \right) = I + 2\,F, \qquad\qquad F^4 = \left(\begin{array}{cc} 5 & 3 \\ 3 & 2 \end{array} \right) \;= 2\,I + 3\,F, \qquad \text{etc....}$$

We also have $C_1 + (1-i)2\sigma_1 - i2\sigma_3 = (1-i)F = \sqrt{2}\, e^{-i\frac{\pi}{4}} F$, which shows the relation between the Chomsky matrix C_1 and the Fibonacci matrix F.

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